Supplementary Material

The sinusoids used in all the simulations are modeled by the following equation:

$$\text{Sine} = B\,\mathit{sin}(2\pi f t)$$

And the Gaussian noise is modeled as

$$A = \sqrt{n}\,\xi(t) \qquad (\xi(t) \text{ is Gaussian noise, } n = 0.1)$$

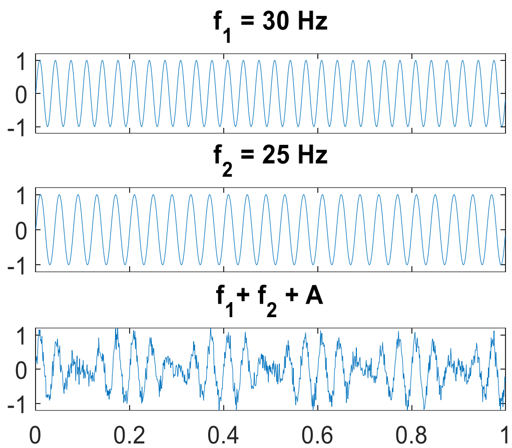

Figure 1 Time domain addition of two pure sinusoids

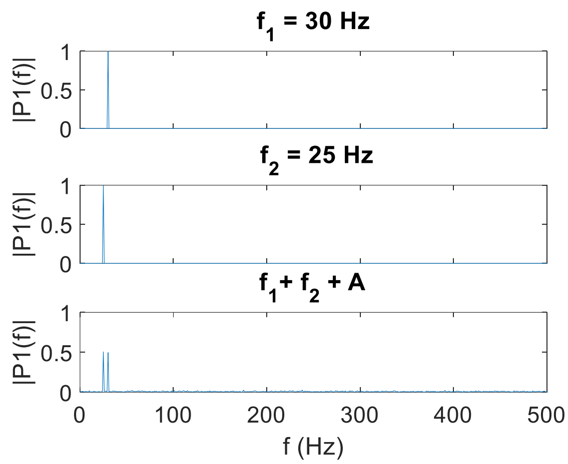

Figure 2 Frequency domain addition of two pure sinusoids

Figure 1 shows the addition of two pure sinusoids (in time domain) having a frequency difference of five. Addition of f_1 and f_2 shows a resultant signal with clearly visible beating effect. The new signal shows five distinct beats in the figure. However, it is hypothesized that the clear visibility of beats is largely due to the simplistic signals used for addition i.e., each sinusoid contains just one frequency.

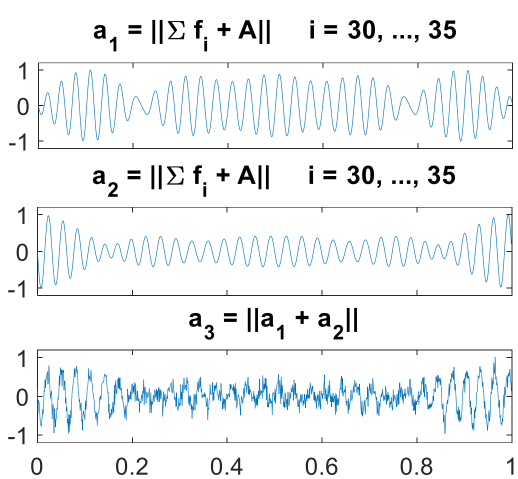

Figure 3 Normalized time domain addition of five variable amplitude sinusoids with random noise

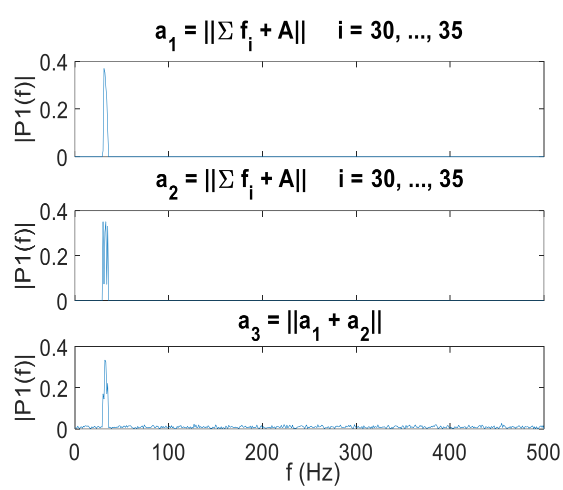

Figure 4 Normalized frequency domain addition of five variable amplitude sinusoids with random noise

Figure 3 shows the addition of two more complex sinusoids. The signals $a_1$and $a_2$are composed of five different sinusoids and Gaussian random noise is added to each of the constituent signals. Both $a_1$and a_2 constitute of the same frequencies but the difference in waveform is caused by the random addition of noise. The signal a_3 is the normalized addition of $a_1$and a_2. It can be seen that beats are far less visible in a_3. It can be inferred that addition of slightly more complex signals ($a_1$and a_2) shows lesser beats.

In Fig. 5 we add two signals, each containing 270 different frequencies having variable magnitudes. Each signal is normalized also. These two signals are added together and inspected for beats. As it is visible from a_3 that the signal doesn't contain any beats due to the presence of a large number of frequencies which increase the complexity of the signal.

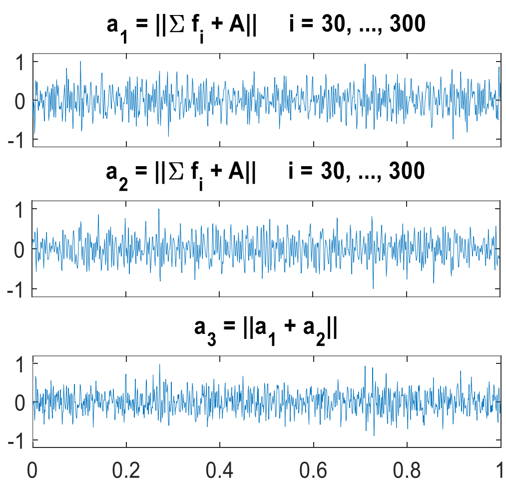

Figure 5 Normalized time domain addition of 270 amplitude sinusoids with random noise

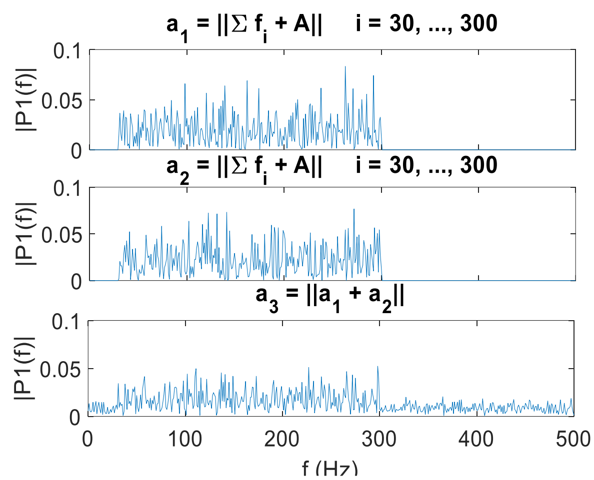

Figure 6 Normalized frequency domain addition of 270 variable amplitude sinusoids with random noise

From the above examples it can be inferred that beats occur most visibly in addition of pure sinusoids and increasing the complexity of a signal helps in decreasing beats. As we continue on adding more and more frequencies together, the beating phenomenon decreases, and the addition of a large number of frequencies with varying amplitudes masks the beating effect.

The signals used in our framework comprise of frequencies in the range of one to 1000, and in addition to that these signals are stochastic in nature. Stochastic signals are aperiodic and non-deterministic signals. The signals are directly dependent on the real time force and velocity input from the users, and as such cannot be predicted in advance. Thus, if we consider the addition of signals used in our framework, given their stochastic nature, it can be argued that the occurrence of beats will be an extremely rare phenomenon.

List of adjectives used in Experiment 2: Adjective Rating

Sticky	Slippery	Flat	Even
Irritating	Rough	Hard	Dense
Bumpy	Pleasant	Sharp	Thick
Uneven	Dull	Sparse	Smooth
Prickly	Thin	Soothing	Soft